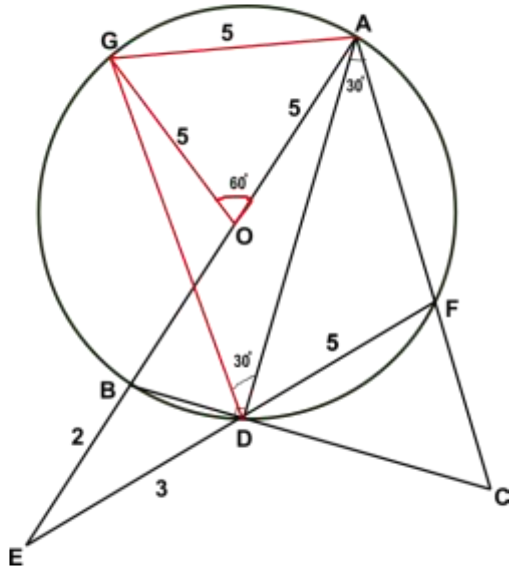
Author's Solution



Given : AB = 10, BE=2 , DF = 5, $\angle DAC = 30°$, AD ⊥ BC

To prove : AF = 2FC

Construction:

Mark the midpoint 'O' of AB. Since ΔADB is right Δ, its circumcentre is O and OA=OB=OD=5. Now draw the circumcircle of ΔADB. Draw DG ∥ AC so as to cut the circle at G. Join OG & AG.

Solution :

Now $\angle ADG = \angle DAF = 30°$ ($\because DG \parallel AF$)

$\therefore \angle AOG = 60°$

$\therefore \angle OAG = \angle OGA = \frac{(180-60)}{2} = 60°$

∴ Δ OAG is equilateral

$\therefore AG = 5$ -------------------- (1)

$\therefore AG = DF = 5$

∴ AGDF is an isosceles trapezium

All isosceles trapeziums are concyclic.

∴ F lies on the circumcircle of $\Delta\, ADG$ & $\Delta\, ADB$.

∴ ABDF is concyclic and the chords AB & FD meet at 'E'

∴ AE x EB = FE x ED

ie 12 x 2 = (5+ED) ED

Solving we get, ED = −8 or 3

ie ED = 3

Now for Δ AEF, BDC is a transversal

$\therefore \frac{AB}{BE} \times \frac{ED}{DF} \times \frac{FC}{CA} = 1$

$\frac{10}{2} \times \frac{3}{5} \times \frac{FC}{CA} = 1$

$\frac{FC}{CA} = \frac{1}{3}$

$\frac{FC}{AF + FC} = \frac{1}{3}$

$\therefore \frac{FC}{AF} = \frac{1}{2}$

ie AF = 2FC ----------------------- Proved

Solution given by
DR. M. RAJA CLIMAX
Founder Chairman
CEOA Group of Institutions

***

COROLLARY: DC = 3BD